DERIVED GEOMETRY LEARNING SEMINAR

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In this learning seminar, we will try to understand some basic derived algebraic geometry, concentrating on the characteristic zero case. In comparison with usual algebraic geometry which is based on commutative algebra and homological algebra, we will take CDGA(commutative differential graded algebra) as the analogue of "commutative algebra" and homotopical algebra as the analogue of "homological algebra". Derived higher stacks will be introduced as our main geometric object of study. The main reference is [1]. Occasionally, more advanced [5, 6, 3] will also be used. If time premits, we could try to sketch some applications.

Lecture 1. Introduction

A quick overview of the topics we want to cover and also discuss some motivating applications of derived geometry. following [1, Section 1].

Lecture 2. Commutative Differential graded algebras

Review the basic definition of CDGA and discuss some basic properties following [1, Section 1].

Lecture 3. Model categories

Review the basic definition of model categories, their homotopy categories, (nonabelian) derived functors and homotopy limits following [1, Section 2].

Lecture 4. The category of CDGA

Introduce the model category of CDGAs and the corresponding homotopy category, Postnikov towers of CDGA as well as derived tensor products following [1, Section 2 ,3].

Lecture 5. Cotangent complex

Discuss tagent spaces and obstruction theories in the derived setting, and then define the cotangent complex following [1, Section 3].

Lecture 6. Some examples: Derived de Rham cohomology and shifted symplectic structures

Discuss the derived de Rham cohomology and shifted symplectic structures following [1, Section 3] and [4].

Lecture 7. Simplicial categories

Introduce to simplicial categories following [1, Section 4] or any book on simplicial categories. e.g., [2].

Lecture 8. Geometric n-stacks

Definition of geometric n-stacks and their basic properties following [1, Section 5.1–2].

Lecture 9. Geometric n-stacks

Quasi-coherent sheaves and hypercomplexes on geometric n-stacks following [1, Section 5.3–4].

Lecture 10. Derived geometric n-stacks I

Basic definitions of derived geometric n-stacks and quasi-coherent complexes on them following [1, Section 6.1–2].

Lecture 11. Derived geometric n-stacks II

Tangent and obstruction theory of derived geometric n-stacks and the cotangent complex on them following [1, Section 6.3–4].

Lecture 12. Derived geometric n-stacks III

Artin-Lurie representability and some more examples following [1, Section 6.5–7].

Lecture 13. Derived deformation theory

A brief sketch of derived deformation theory.

References

[1] J. Eugster and J.P. Pridham, *An introduction to derived geometry*, avaiable online.

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[3] J. Lurie, *Derived Algebraic Geometry V: Structured spaces*, avaiable online.

[4] B. Toen, *Derived algebraic geometry*, EMS Surv. Math. Sci., 2014.

[5] B.Toen and G.Vezzosi, *Homotopical algebraic geometry. I. Topos Theory*, Adv. Math., 2005.

[6] B.Toen and G.Vezzosi, *Homotopical algebraic geometry. II. Geometric stacks and applications*, Mem. Amer. Math. Soc. 193 (2008).

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