

Homework10

Problem 1 Relativistic invariance

Prove that $\vec{E} \cdot \vec{B}$ and $E^2 - c^2 B^2$ are invariant under Lorentz transformation of the electromagnetic field.

Problem 2 An electromagnetic wave

Here is a particular electromagnetic field in free space:

$$\begin{aligned} E_x &= 0, & E_y &= E_0 \sin(kx + \omega t), & E_z &= 0; \\ B_x &= 0, & B_y &= 0, & B_z &= -(E_0/c) \sin(kx + \omega t). \end{aligned}$$

- (a) Show that this field can satisfy Maxwell's equations if ω and k are related in a certain way.
- (b) Suppose $\omega = 10^{10} \text{ s}^{-1}$ and $E_0 = 1 \text{ kV/m}$. What is the wavelength? What is the energy density in joules per cubic meter, averaged over a large region? From this calculate the power density, the energy flow in joules per square meter per second.

Problem 3 Traveling and standing waves

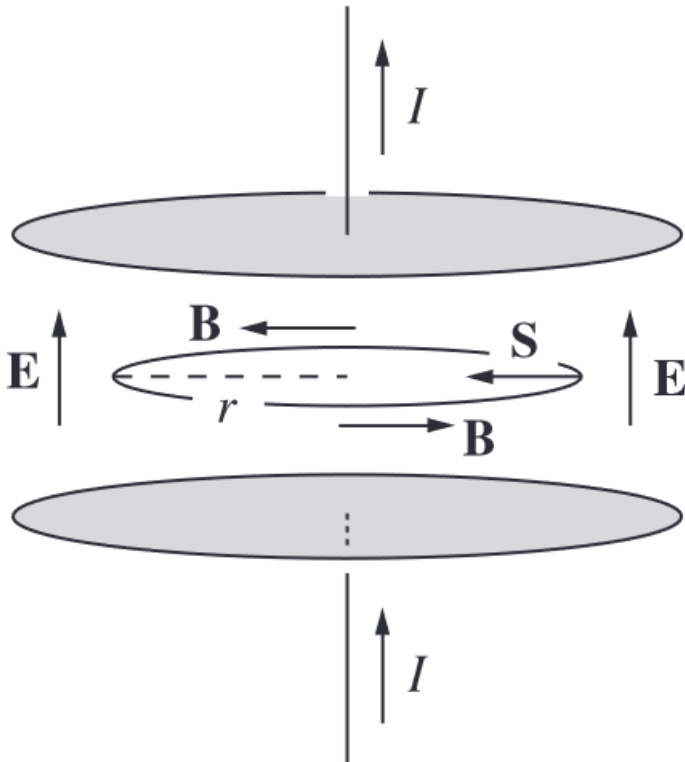
Consider the two oppositely traveling electric-field waves, $\vec{E}_1 = \hat{x} E_0 \cos(kz - \omega t)$ and $\vec{E}_2 = \hat{x} E_0 \cos(kz + \omega t)$. The sum of these two waves is the standing wave, $2\hat{x} E_0 \cos kz \cos \omega t$.

- (a) Find the magnetic field associated with this standing electric wave by finding the \vec{B} fields associated with each of the above traveling \vec{E} fields, and then adding them.
- (b) Find the magnetic field by instead using Maxwell's equations to find the \vec{B} field associated with the standing electric wave, $2\hat{x} E_0 \cos kz \cos \omega t$.

Problem 4 Energy flow into a capacitor

A capacitor has circular plates with radius R and is being charged by a constant current I . The electric field E between the plates is increasing, so the energy density is also increasing. This implies that there must be a flow of energy into the capacitor. Calculate the Poynting vector at radius r inside the capacitor (in terms of r and E), and verify that its flux equals the rate of change of the energy stored in the region

bounded by radius r .



Problem 5 Field momentum of a moving charge

Consider a charged particle in the shape of a small spherical shell with radius a and charge q . It moves with a nonrelativistic speed \vec{v} . The electric field due to the shell is essentially given by the simple Coulomb field, and the magnetic field is then given by $\vec{B} = \frac{\vec{v}}{c^2} \times \vec{E}$. Using the results from Lec. Note 15 section 4, integrate the momentum density over all space. Show that the resulting total momentum of the electromagnetic field can be written as $m\vec{v}$, where $m \equiv (4/3)(q^2/8\pi\epsilon_0 a)/c^2$.