Homework6

Problem 1

By using the relativistic velocity addition law, please directly prove that the $4-momentum$ defined as $(\vec{p}, p^0)$

$$\vec{p} = \frac{m_0 \vec{v}}{\sqrt{1-(v/c)^2}}, p^0 = \frac{m_0 c}{\sqrt{1-(v/c)^2}}$$

satisfies the Lorentz transformation

$$\begin{pmatrix} p'^{,1} \\ p'^{,0} \end{pmatrix} = \begin{pmatrix} \gamma & -\gamma\beta \\ -\gamma\beta & \gamma \end{pmatrix} \begin{pmatrix} p^1 \\ p^0 \end{pmatrix}$$

We suppose that the frame F' moves along the x-direction at the velocity u relative the the frame of F . $\beta = u/c$ and $\gamma = 1/\sqrt{1-\beta^2}$. The quantities with a prime are the ones measured in the frame F', and those without a prime are measured in the frame F.

You may need to use the following relation.

$$1 - \left(\frac{v'}{c}\right)^2 = \frac{\left(1-\frac{u^2}{c^2}\right)\left(1-\frac{v^2}{c^2}\right)}{\left(1-\frac{\vec{u}\cdot\vec{v}}{c^2}\right)^2}$$

You are also encouraged to prove this.

Problem 2 Galactic velocities

We observe a galaxy receding in a particular direction at a speed $V = 0.3c$, and another galaxy receding in the opposite direction with the same speed. What speed of recession would an observer in one of these galaxies observe for the other galaxy?

Problem 3 Wavevector and frequency

Prove that the wavevector $\vec{k}$ and frequency ω of a propagating wave combined in the way of

$$k^\mu = (\vec{k}, k^0 = \frac{\omega}{c})$$

satisfy 4-vector according to the Lorentz transformation.
Please prove it in two di erent ways.

1. You may use the fact that the phase di erence over a space-time interval $(\Delta\vec{x}, t)$, i.e, $\vec{k}\cdot\Delta\vec{x} - \omega\Delta t$ is a Lorentz invariant to prove the above result.
2. Consider a special case that $\vec{k}$ is along the x-direction, and the relative motion between F' and F frame is also along x-direction. You may examine how to define wave-lengths and periods in these two different frames.